Mind Game # 2 - Detailed Solution, Comments and Winners

Mind Game # 2

1	32
2	64
4	128
8	256
16	512
32	1024

How many different positive integer multiples of 32, including 1 by 32 and 42 by 32, can be calculated using the above two columns and the ancient Egyptian method of multiplication?

Solution

We have to find out the number of different positive integers that can be formed as a sum of numbers in the left hand column where each number is used only once. Such numbers can be written as non-zero binary numbers consisting of at most 6 digits. For example in binary the sum of 2, 8 and 16 is 11010, the sum of 1, 2, 8 and 32 is 101011and the sum of 1, 2, 4, 8, 16 and 32 is 111111. Also, each non-zero binary number consisting of at most 6 digits corresponds to a sum of numbers in the left hand column where each number is used only once. Now, how many binary numbers are there from 1 to 111111 inclusive of 1 and 111111? There are $2^6 - 1 = 63$ numbers. Now, we have to show that these 63 numbers are different. This can be shown by using the fact that $32 < 16 + 8 + 4 + 2 + 1$, $16 < 8 + 4 + 2 + 1$, $8 < 4 + 2 + 1$, $4 < 2 + 1$ and $2 < 1$. Any two of these 63 numbers differ in at least one place value. Let's look at the highest such place value. Then one number has 1 in this place and the other number has 0 in this place. So the number with 1 in the highest place where two numbers differ is greater than the other number. So 63 different positive integer multiples of 32, including 1 by 32 and 42 by 32, can be calculated using the above two columns and the ancient Egyptian method of multiplication. □

Comments

1. This problem can be solved without using the binary notion. How many different sums using the numbers in the left hand column where each number is used only once can be formed? This is equal to the number of non-empty subsets of {1, 2, 4, 8, 16, 32}. This can be found using GCE AL combinatorics. The number of non-empty subsets is ${}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$. Again we have to show that sums corresponding to each non empty subset are different.
2. None of the solutions received has shown that sums are different. Some who sent solutions assumed that numbers in the left hand column can be used more than once. But in the ancient Egyptian method of multiplication each number, or row, is used only once.

Winners

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2. Warnakulasuriya Don Emmanuel Hashika
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   Age: 19
   A/L 2019 – Private

4. V.G.T.Aravindi Epa
   Ariya Niwasa,Vidyachandra Mawatha, Ahangama
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   Southlands College, Galle
   Advanced level

5. Amila Abeysinghe
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   Combined Mathematics Teacher

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