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SCHOOL OF ACCOUNTING AND BUSINESS BSc. (APPLIED ACCOUNTING) GENERAL / SPECIAL DEGREE PROGRAMME

## YEAR II SEMESTER II END SEMESTER EXAMINATION - AUGUST 2015

## QMT 20330 Operational Research

| Date | $:$ | $12^{\text {th }}$ August 2015 |
| :--- | :--- | :--- |
| Time | $:$ | 5.30 p.m. -8.30 p.m. |
| Duration | $:$ | Three $(03)$ hours |

Instructions to Candidates:

- Answer ONLY FIVE (05) questions.
- The total marks for the paper is 100 .
- All questions carry equal marks.
- Use of scientific calculator is allowed.
- Standard Normal Table and Formula Sheets are provided.
- Answers should be written neatly and legibly.


## Question No. 01

A cargo plane has three compartments for storing cargo: front, centre and rear. These compartments have the following limits on both weight and space:

| Compartment | Weight capacity (tons) | Space capacity (cubic meters) |
| :--- | :---: | :---: |
| Front | 10 | 6800 |
| Centre | 16 | 8700 |
| Rear | 8 | 5300 |

Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the plane.

The following four cargoes are available for shipment on the next flight:

| Cargo | Weight (tons) | Volume (cubic meters/ton) | Profit (£/ton) |
| :--- | :---: | :---: | :---: |
| C1 | 18 | 480 | 310 |
| C2 | 15 | 650 | 380 |
| C3 | 23 | 580 | 350 |
| C4 | 12 | 390 | 285 |

Any proportion of these cargoes can be accepted. The objective is to determine how much (if any) of each cargo C1, C2, C3 and C4 should be accepted and how to distribute each among the compartments so that the total profit for the flight is maximized.
i. Formulate the above problem as a linear program
ii. What assumptions are made in formulating this problem as a linear program?
iii. Briefly describe the advantages of using a software package to solve the above linear program, over a judgmental approach to this problem.

## Question No. 02

You are the hospital gardener. The flowers require 28 pounds of nitrogen, 30 pounds of phosphates, and 4 pounds of potash. You have two fertilizers to use. One kind costs $\$ 4$ per bag. It contains 4 pounds of nitrogen, 2 pounds of phosphates, and no potash per bag. The other kind costs $\$ 5$ per bag. It contains 2 pounds of nitrogen, 3 pounds of phosphates, and 1 pound of potash.
i. How many bags of each kind of fertilizer do you use to minimize your costs?
ii. The CEO's brother owns the company that makes the first kind of fertilizer. You must buy at least 6 bags of it. Now how much of each kind of fertilizer do you use?

## Question No. 03

The APEX Company is the distributor for television receivers. It owns three warehouses with the following stocking capacity:

| Warehouse Location | Sets in Stock |
| :---: | :---: |
| A | 100 |
| B | 25 |
| C | 75 |

Further the APEX Company has the following order for set deliveries:

| Market location | Orders |
| :---: | :---: |
| X | 80 |
| Y | 30 |
| Z | 90 |

The delivery costs form warehouses to each market location are largely a function of mileage or distance. From the past experience the company has determined the per unit transportation costs and summarized in the below given table:

|  | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ |
| :--- | :--- | :--- | :--- |
| $\mathbf{A}$ | 5 | 10 | 2 |
| $\mathbf{B}$ | 3 | 7 | 5 |
| $\mathbf{C}$ | 5 | 8 | 4 |

The deliveries could be made in many ways but the company would like to deliver the T.V. sets at the minimum cost. Since you are the assistant to the transportation manager of the APEX Company, the manager asks you to come up with the distribution schedule that would minimize the delivery cost of the company.

Note: Choose the methods that are efficient

## Question No. 04

Our observations of one cashier in a super market have shown that the arrival distribution of customers follows a Poisson distribution with an arrival rate of $3 / 4$ customers per 5 minutes. The distribution of service time follows an exponential distribution with an average service time of 1079/1000 minutes per customer. Calculate all the measurements of effectiveness of this queuing system and interpret.

Note: Clearly indicate the units of the measurements if applicable

## Question No. 05

The table below gives precedence relationships, the durations and the crashing costs of various activities of a construction project. Further the last two columns gives the crashing cost of each activity on day one and day two respectively

| Successor <br> activity | Predecessor <br> activity | Duration (days) |  |  | Crashing Cost |  |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathbf{t}_{\mathbf{m}}$ | $\mathbf{t}_{\mathbf{p}}$ | First day | Second day |  |
| A | - | 06 | 7.5 | 12 | 15000 | 20000 |
| B | - | 07 | 10 | 12 | 10000 | 10000 |
| C | A | 20 | 22 | 25 | 5000 | 5000 |
| D | B | 14 | 15 | 17 | 20000 | 21000 |
| E | A | 10 | 12 | 15 | 16000 | 18000 |
| F | C, D | 10 | 12 | 14 | 12000 | 15000 |
| G | B | 12 | 14 | 18 | 24000 | 24000 |
| H | E | 16 | 18 | 21 | - | - |
| I | C, D, G | 12 | 14 | 17 | 30000 | - |
| J | E | 01 | 02 | 03 | 25000 | 25000 |
| K | F, H, I | 07 | 09 | 11 | 10000 | 10000 |
| L | C, D, G | 17 | 19 | 22 | - | - |
| M | J | 07 | 08 | 10 | 40000 | - |

i. Based on the above table draw the PERT network
ii. Calculate the necessary measures (Only the Average Durations should be rounded off to the nearest integer during the calculations)
iii. Identify the critical activities
iv. What is the probability that the project duration will exceed by not more than 5 days from the expected project completion time.
v. Due to various reasons the project manager needs to complete the project four days before the expected project completion time. Since you are the assistant to him he asks you to prepare the crashing schedule

## Question No. 06

i. A computer system can operate in two different modes. Every hour, it remains in the same mode or switches to a different mode according to the transition probability matrix

$$
P=\left[\begin{array}{ll}
0.4 & 0.6 \\
0.6 & 0.4
\end{array}\right]
$$

a. Compute the 2 -step transition probability matrix.
b. If the system is in Mode I at $5: 30 \mathrm{pm}$, what is the probability that it will be in Mode I at $8: 30 \mathrm{pm}$ on the same day?
ii. Consider a game of "ladder climbing". There are 5 levels in the game, level 1 is the lowest (bottom) and level 5 is the highest (top). A player starts at the bottom. Each time, a fair coin is tossed. If it turns up heads, the player moves up one rung. If tails, the player moves down to the very bottom. Once at the top level, the player moves to the very bottom if a tail turns up, and stays at the top if head turns up.
a. Find the transition probability matrix.
b. Find the steady-state distribution of the Markov chain.

## Formula Sheet

$$
\begin{array}{ll}
t=\frac{a+4 m+b}{6} & \rho \\
\sigma_{i}=\frac{b-a}{6} & P_{o} \\
\sigma=\sqrt{\sum \sigma_{i}^{2}} & P_{n} \\
z=\frac{X-\mu}{\sigma} & P_{o}\left(\frac{\lambda}{\mu}\right)^{n} \\
L_{q} & =\frac{\lambda}{\mu(\mu-\lambda)} \\
L_{s} & =\frac{\lambda^{2}}{\mu-\lambda} \\
W_{q} & =\frac{\lambda}{\mu(\mu-\lambda)} \\
W_{s} & =\frac{1}{(\mu-\lambda)}
\end{array}
$$

## Standard Normal Table

|  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.49865 | 0.49869 | 0.49874 | 0.49878 | 0.49882 | 0.49886 | 0.49889 | 0.49893 | 0.49896 | 0.49900 |
| 3.1 | 0.49903 | 0.49906 | 0.49910 | 0.49913 | 0.49916 | 0.49918 | 0.49921 | 0.49924 | 0.49926 | 0.49929 |
| 3.2 | 0.49931 | 0.49934 | 0.49936 | 0.49938 | 0.49940 | 0.49942 | 0.49944 | 0.49946 | 0.49948 | 0.49950 |
| 3.3 | 0.49952 | 0.49953 | 0.49955 | 0.49957 | 0.49958 | 0.49960 | 0.49961 | 0.49962 | 0.49964 | 0.49965 |
| 3.4 | 0.49966 | 0.49968 | 0.49969 | 0.49970 | 0.49971 | 0.49972 | 0.49973 | 0.49974 | 0.49975 | 0.49976 |
| 3.5 | 0.49977 | 0.49978 | 0.49978 | 0.49979 | 0.49980 | 0.49981 | 0.49981 | 0.49982 | 0.49983 | 0.49983 |

