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THE INSTITUTE OF CHARTERED ACCOUNTANTS OF SRI LANKA

SCHOOL OF ACCOUNTING AND BUSINESS
BSc. (APPLIED ACCOUNTING) GENERAL / SPECIAL DEGREE PROGRAMME

YEAR I SEMESTER I (Intake IV- Group B)

END SEMESTER EXAMINATION – SEPTEMBER 2015

QMT10130 Business Mathematics

Date	:	18th September 2015
Time	:	5.30 p.m. - 8.30 p.m.
Duration	:	Three (03) Hours

Instructions to Candidates:

- Answer ALL questions.
- Allocated marks for each question is indicated.
- Total marks for the paper is 100.
- Use of non-programmable electronic calculator is allowed.
- Formula sheet will be provided.
- Answers should be written clearly with the required steps.

Question No. 01

A manufacturing firm producing components has a fixed cost of Rs. 12,000 per day and a variable cost of Rs. 160 per component. When ***q*** components are produced and sold per day the demand function is ***q = 800-2p***, where ***p*** is the selling price per component in rupees.

a. Show that the total cost function per day is $C(q) = 12{,}000 + 160q$

(1 Marks)

b. Show that the total revenue function per day is $R(q) = 400q - \frac{q^2}{2}$

(Hint : Total revenue per day equals to total number of components sold per day multiplied by the selling price per component)

(2 Marks)

c. Write down the Profit function P(**q**) per day.

(2 Marks)

d. Find the production level per day which maximizes the profit. (Hint use first and second order derivatives with respect to **q**)

(4 Marks)

e. Show that the Marginal Cost is equal to the Marginal Revenue at the Maximum Profit level of production.

(Hint: Marginal Cost is $\frac{dC(q)}{dq}$ and Marginal Revenue is $\frac{dR(q)}{dq}$)

(2 Marks)

f. Find the Maximum Profit that the firm can expect per day.

(2 Marks)

g. What is the selling price per component at the maximum profit per day

(2 Marks)

h. Find the cost incurred and revenue earned at the maximum profit per day.

(2 Marks)

(Total 17 Marks)

Question No. 02

A firm produce two types of items namely Type ***R*** and Type ***S.*** The number of units produced, unit selling price and unit cost of each type of item are given in the following table.

	Type ***R***	Type ***S***
No of units produced	***X***	***Y***
Unit selling price	210 – ***X***	805 – 8***Y***
Unit cost	10	5

Assuming that all the items produced from Type ***R*** and Type ***S*** are sold, answer the following questions.

a. Find the total revenue function R(X,Y) (i.e summation of total revenue of Type *R* and total revenue of Type *S*)

(3 Marks)

b. Find the total cost function C(X,Y) (i.e summation of total cost of Type *R* and total cost of Type *S*)

(3 Marks)

c. Write down the Profit function P(X, Y).

(2 Marks)

d. Find the number of items of each type that should be produced to maximize the profit. (Hint : use first and second order partial derivatives)

(4 Marks)

e. Find the maximum profit.

(2 Marks)

f. Given the function $U(x,y) = x^4 + x^2y^2 + y^4$ show that $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = 4U$

(3 Marks)

(Total 17 Marks)

Question No. 03

I. The demand and supply functions in a perfect competition are given by $\mathbf{P_d = 64\text{-}q^2}$ and $\mathbf{P_s = 3q^2}$ respectively. Where $\mathbf{P_d, P_s}$ and **q** represent demand price, supply price and quantity demanded or supplied respectively.

a. Find the equilibrium quantity and the equilibrium price.

(2 Marks)

b. Sketch the demand and supply functions for q >= 0, clearly showing all the required points.

(2 Marks)

c. Highlight and name the consumer surplus (CS) and producer surplus (PS) in the above sketch.

(2 Marks)

d. Find the Consumers' Surplus (CS)

(3 Marks)

e. Find the Producers' Surplus (PS)

(3 Marks)

II. If the marginal revenue of a commodity is given by $MR(q) = 6q^2 + 2q - 1$ where the $\boldsymbol{q}$ is the quantity supplied. Find the revenue function given that total revenue is zero when quantity supplied is zero (***q=0***). Also find the unit selling price function *UP(q)*. Further show that at least one unit of commodity need to be sold in order for the unit selling price to be greater than zero. (Hint MR = $= \frac{dR}{dq}$, Revenue = Unit selling price × no of units sold)

(5 Marks)

(Total 17 Marks)

Question No. 04

I. A nurse is responsible for buying week's supply of food and medication for the boys and girls at a local nursing center. The food and medication for each boy costs twice as much as those supplies for a girl. The nurse needs to feed 32 girls and 24 boys. Provided she has Rs 1,600 for this purpose.

a. Develop a system of simultaneous equations representing the problem using the identified unknowns.

(3 Marks)

b. Find the amount that the nurse can spend for each girl and boy for food and medication by solving above system of simultaneous equations.

(2 marks)

II. A component of an item produce has the shape of a right-angled triangle such that its sides are ***x** cm*, **x+*1*** *cm* and ***5*** *cm*. Find the area of the triangle if the longest side (i.e. hypotenuse) is ***5*** *cm*. (Hint: for a right-angled triangle with sides a, b and c, where c is the hypotenuse Pythagoras relationship $c^2 = a^2 + b^2$ holds.)

(5 Marks)

III. Two tubes, working together, can fill the reservoir with the liquid in 12 hours. The larger tube, if works separately, can fill the reservoir in 18 hours faster than the smaller tube. Find the time to fill the reservoir using the smaller tube.

(5 Marks)

(Total 15 Marks)

Question No. 05

I. The 4th and 8th term of a geometric progression are $\frac{1}{16}$ and $\frac{1}{256}$ respectively.

a. Find the first term and the common difference.

(2 Marks)

b. Find the sum of the first 11 terms to the nearest 6 decimals.

(2 Marks)

c. Find the 10th Term

(2 Marks)

II. A company has allocated Rs 5,900 for miscellaneous expenses for the proposed extension of ~~to~~ its factory floor. On the first day the company has spent 50 rupees and every subsequent day the expense is increased by an additional 5 rupees than the previous day. How long the company can survive with the available budget.

(4 Marks)

III. Let $f(x) = x^2 - 11x + 30$, find the following.

a. Gradient of the tangent to the curve at the point (2,12)

(3 Marks)

b. Find the equation of the tangent at the point (2,12)

(2 Marks)

c. Coordinate of the point where the equation of the tangent intersect the X-axis

(2 Marks)

(Total 17 Marks)

Question No. 06

I. Find the effective rate of interest for a nominal rate of 12% per annum compounded quarterly.

(3 Marks)

II. A debt of 75,000 which is due in 7 years from now is to be paid off by three payments Rs. 20,000 now, Rs 12,500 after 4 years and a final payment at the end of 6 years. Find the final payment assuming an interest rate of 12% per annum.

(4 Marks)

III. An annuity consisting of equal payments at the end of each quarter for 3 years is to be purchased for Rs. 10,000. If the interest rate is 10% per annum compounded quarterly, find the value of each payment.

(4 Marks)

IV. A machine with an original cost of Rs. 50,000 has an estimated salvage value of Rs. 2,000 after 6 years. Assuring an interest rate of 5% per annum, find the annual deposit to be made to a sinking fund.

(3 Marks)

V. Find the present value of a perpetuity of Rs. 1,800 per annum if money worth 12% compounded annually.

(3 Marks)

(Total 17 Marks)

Formula Sheet

$nth\ term\ of\ a\ arithmetic\ series\ T_n = a + (n-1)d$

$sum\ of\ n\ terms\ of\ a\ arithmetic\ series\ Sn = \frac{n}{2}[2a + (n-1)d]$

$nth\ term\ of\ a\ geometric\ series\ \ T_n = ar^{n-1}$

$sum\ of\ n\ terms\ of\ a\ geometric\ progression\ when\ r > 1:\ S_n\ = \frac{a\ (\ r^n - 1)}{r-1}$

$sum\ of\ n\ terms\ of\ a\ geometric\ progression\ when\ r < 1:\ S_n\ = \frac{a\ (1 -\ r^n)}{1-r}$

$roots\ of\ the\ quadratic\ equation\ ax^2 + bx + c = 0\ where\ a \neq 0\ is\ \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$Definite\ integral\ of\ x^n\ \ :\ \ \int_b^a x^n\ dx = [\frac{x^{n+1}}{n+1}]_b^a$

$Compounding\ FV\ = PV(\ 1 + r\ )^n$

$Discounting\ PV = \frac{FV}{(1+r)^n}$

$$PV_{ODA} =\ A\ \left[\frac{1 -\ (1+r)^{-n}}{r}\right]$$

$$PV_{DueA} =\ A\ \left[\frac{1 -\ (1+r)^{-n}}{r}\right](1+r)$$