SCHOOL OF ACCOUNTING AND BUSINESS
BSc. (APPLIED ACCOUNTING) GENERAL / SPECIAL DEGREE
PROGRAMME

YEAR I SEMESTER I (Intake V – Group A)
END SEMESTER EXAMINATION – FEBRUARY 2016

QMT 10130 Business Mathematics

Date	:	12th February 2016
Time	:	9.00 a.m. – 12.00 p.m.
Duration	:	Three (03) hours

Instructions to Candidates:

- Answer only **FIVE (05)** questions.
- All questions carry equal marks.
- The total marks for the paper is 100.
- Formula Sheet is provided.
- Use of scientific calculator is allowed.
- Answers should be written neatly and legibly

Question No. 01

The cost and revenue functions of a company which produces a particular heavy duty tool are given by.

$$TR = 2000\,Q - 45Q^2 \text{ and}$$
$$TC = 1500 - 100\,Q^2 + Q^3$$

Where Q is the number of units produced. Assume that all the units produced are sold without any restriction.

   i. Find the fixed cost.

   ii. Set up the profit function, π, for the given company.

   iii. Find the derivative of the profit function with respect to the appropriate variable.

   iv. Find the critical points of the profit function, π.

   v. Find the appropriate second order derivative.

   vi. Find the number of units that should be produced to maximize the profits.

   vii. Find the maximum profit.

      Hint: If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Question No. 02

   i. If $U(x,y) = 2x^3y + 3x^2y^2 + 4xy^3$, without using Euler's theorem prove that

$$x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = 4\,U$$

   ii. The profit function $P(x,y) = 22x + 22y - 0.04x^2 - 0.10y^2 - 1550$ of a firm is assumed to have a monopoly on x and y, but the markets do not intersect

      a. Find the two first order partial derivatives P_x and P_y.

      b. Set P_x and P_y equal to zero and solve for x and y.

      c. Find the second order partial Derivatives
$$P_{xx}, P_{yy} \text{ and } P_{xy}.$$

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d. Evaluate the second order partial derivatives at the critical points obtained in part (b).

e. Show that the following condition holds at the critical point.

$$P_{xx}\ P_{yy} > \left[P_{xy} \right]^2$$

f. Since the above condition is satisfied, the critical point is a maximum point for the profit. Find the maximum profit?

Question No. 03

i. A sum of Rs. 75,000 was invested in a bank account at an interest rate of 10% compounded quarterly. Five years later the rate decreased to 6% compounded semiannually. If no withdrawals are made, what would be the balance in the account at the end of 12 years after the deposit was made?

ii. A company buys a machine for Rs. 170,000. If it is assumed to depreciate at a fixed rate of 12% per annum, what will its value be after 5 years?

iii. A company needs to replace a machine costing Rs. 550,000 in 10 years. To achieve this it will make annual investments, for ten years starting immediately, at a rate of 7.5% per annum. Find the amount of the annual investment to achieve this target.

Question No. 04

i. A Rs. 250,000 mortgage is taken out on a property at a rate of 10 percent for 20 years. What will the monthly repayment be? After 10 years of the mortgage, the interest rate increases to 12 percent, recalculate the monthly repayment figure.

   Hint: You are allowed to round of the monitory figures to the nearest rupee during calculations

ii. A credit card charges an annual rate of 18% compounded monthly. This month's bill is Rs. 35,000... Suppose that you keep paying Rs. 5000 each month. How long will it take to pay off the bill? What is the total interest paid during that period?

iii. Over 10 years a bond costing Rs. 3000 increases in value to Rs. 5372.54. Find the effective annual rate.

Question No. 05

i. The demand and supply functions under pure competition are given by $p_d = 490 - 10\,q^2$ and $p_s = 315 + 3q^2$ respectively. Where p and q are the price and the quantity.

   a. Find the equilibrium quantity and the equilibrium price.
   b. Sketch the demand and the supply functions clearly stating all the required points.
   c. Highlight the consumer and producer surpluses and indicate in the sketch.
   d. Find the consumers' surplus.
   e. Find the producers' surplus.

ii. If the marginal revenue of a commodity is given by $MR = 600 + 65\,q - 2\,q^2$, Find the total revenue function and deduce the demand function

Question No. 06

i.　If $A = \begin{pmatrix} -2 & 1 \\ 5 & -6 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 0 \\ 6 & 7 \end{pmatrix}$

Find the matrix X which satisfies the following relationship $4A + 5B - 2X = 0$

ii. Ms. Amila invested Rs. 1,000,000 in two different investments. A part of the money is invested at 9%, and the rest at 6%. The total interest earned through the investment during the last year was Rs. 81,000.

   a. Identify the unknowns to be evaluated in the above problem.

   b. Develop the system of simultaneous equations which represent the above problem.

   c. Find the amounts invested at 9% and 6% separately through the solution of the system of simultaneous equation you developed in part (b) using the matrix method.

FORMULA SHEET

$$V = P(1 + r\,n)$$

$$V = P(1 + r)^n$$

$$V = P(1 - r)^n$$

$$P_{ODI} = R \left\{ \frac{1 - (1 + r)^{-n}}{r} \right\}$$

$$A_{ODI} = R \left\{ \frac{(1 + r)^n - 1}{r} \right\}$$

$$P_{PER} = R \left\{ \frac{1}{r} \right\}$$

$$IRR = r_1 + \left\{ \frac{NPV_1}{NPV_1 - NPV_2} \right\} (r_2 - r_1)$$

$$A^{-1} = \left\{ \frac{1}{|A|} \right\} adj(A)$$

$$AX = b \Rightarrow X = A^{-1}b$$