No. of Pages - 05
No of Questions - 06

THE INSTITUTE OF CHARTERED ACCOUNTANTS OF SRI LANKA

SCHOOL OF ACCOUNTING AND BUSINESS
BSc. (APPLIED ACCOUNTING) GENERAL / SPECIAL DEGREE PROGRAMME 2013/14

YEAR I SEMESTER I (Group B)
END SEMESTER EXAMINATION - AUGUST 2014

## QMT 10130 Business Mathematics

| Date | $:$ | $01^{\text {st }}$ August 2014 |
| :--- | :--- | :--- |
| Time | $:$ | 05.30 p.m. -8.30 p.m. |
| Duration | $:$ | Three (03) hours |

## Instructions to Candidates:

- Answer ALL questions.
- The total marks for the paper is 100 .
- All questions carry equal marks.
- Use of non-programmable electronic calculators is allowed.
- Formula Sheet is provided.
- Answers should be written neatly and legibly.


## Question No. 01

The cost and revenue functions of a company which produces a particular heavy duty tool are given by.

$$
\begin{aligned}
& T R=3000 Q-5 Q^{2} \text { and } \\
& T C=Q^{3}-50 Q^{2}+20000
\end{aligned}
$$

Where $\boldsymbol{Q}$ is the number of units produced. Assume that all the units produced are sold without any restriction.
i. Find the fixed cost.
ii. Set up the profit function, $\pi$, for the given company.
iii. Find the derivative of the profit function with respect to the appropriate variable.
$i v$. Find the critical points of the profit function, $\pi$.
v. Find the appropriate second order derivative.
vi. Find the number of units that should be produced to maximize the profits.
vii. Find the maximum profit.

Hint: If $a x^{2}+b x+c=0$ then $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

## Question No. 02

i.If $\boldsymbol{U}(x, y, z)=5 x^{4} y z+9 x y^{4} z+2 y^{6}$, without using Euler's theorem prove that

$$
x \frac{\partial U}{\partial x}+y \frac{\partial U}{\partial y}+z \frac{\partial U}{\partial z}=6 U
$$

ii. The profit function $\boldsymbol{P}(\boldsymbol{x}, \boldsymbol{y})=\mathbf{1 1 x}+\mathbf{1 1} \boldsymbol{y}-\mathbf{0 . 0 2 x ^ { 2 }}-\mathbf{0 . 0 5} \boldsymbol{y}^{\mathbf{2}} \mathbf{- 7 7 5}$ of a firm is assumed to have a monopoly on $x$ and $y$, but the markets do not intersect
a. Find the two first order partial derivatives $P_{x}$ and $P_{y}$.
b. Set $P_{x}$ and $P_{y}$ equal to zero and solve for $x$ and $y$.
c. Find the second order partial Derivatives

$$
P_{x x}, P_{y y} \text { and } P_{x y}
$$

d. Evaluate the second order partial derivatives at the critical points obtained in part (b).
e. Show that the following condition holds at the critical point.

$$
P_{x x} P_{y y}>\left[P_{x y}\right]^{2}
$$

f. Since the above condition is satisfied, the critical point is a maximum point for the profit. Find the maximum profit?

## Question No. 03

i. A sum of Rs. 50000 was deposited in a bank at an interest rate of $13 \%$ compounded quarterly. Five years later the rate decreased to $8 \%$ compounded semiannually. If the money was not withdrawn, what would be the balance in the account at the end of 12 years after the deposit was made?
ii. A company buys a machine for Rs. 50,000 . If it is assumed to depreciate at a fixed rate of 12 per cent per annum, what will its value be after 5 years?
iii. A company needs to replace a machine costing Rs. 150,000 in 7 years. To achieve this it will make seven annual investments, starting immediately, at 7 percent. Find the value of the annual investment to achieve this target.

## Question No. 04

i. A Rs. 200000 mortgage is taken out on a property at a rate of 10 percent for 20 years. What will the monthly repayment be? After 10 years of the mortgage, the interest rate increases to 12 percent, recalculate the monthly repayment figure.

Hint: You are allowed to round of the monitory figures to the nearest rupee during calculations
ii. A credit card charges an annual rate of $12 \%$ compounded monthly. This month's bill is Rs. 22000. The minimum payment is Rs. 3000. Suppose that you keep paying Rs. 3000 each month. How long will it take to pay off the bill? What is the total interest paid during that period?
iii. Over 10 years a bond costing Rs. 3000 increases in value to Rs. 5372.54. Find the effective annual rate.

## Question No. 05

i. The demand and supply functions under pure competition are given by $\boldsymbol{p}_{\mathrm{d}}=\mathbf{4 5}-\mathbf{q}^{2}$ and $\boldsymbol{p}_{\mathrm{s}}=\mathbf{3} \boldsymbol{q}^{\mathbf{2}}+\mathbf{9}$ respectively. Where $\boldsymbol{p}$ and $\boldsymbol{q}$ are the price and the quantity.
a. Find the equilibrium quantity and the equilibrium price.
b. Sketch the demand and the supply functions clearly stating all the required points.
c. Highlight the consumer and producer surpluses and indicate in the sketch.
d. Find the consumers' surplus.
e. Find the producers' surplus.
ii. If the marginal revenue of a commodity is given by $\boldsymbol{M R}=\mathbf{9 - 6} \boldsymbol{x}^{2}+\mathbf{2 x}$, Find the total revenue function and deduce the demand function

## Question No. 06

i. If $\quad A=\left(\begin{array}{rr}-1 & 2 \\ 3 & -4\end{array}\right)$ and $B=\left(\begin{array}{rr}-1 & 0 \\ 2 & 3\end{array}\right)$

Find the matrix $X$ which satisfies the following relationship $4 A+5 B-2 X=0$
ii. Ms. Maheshi invested Rs. 300,000: part at 5\%, and part at $8 \%$. The total interest on the investment was Rs. 21,000.
a. Identify the unknowns to be evaluated in the above problem.
b. Develop the system of simultaneous equations which represent the above problem.
c. Find the amounts invested at $5 \%$ and $8 \%$ separately through the solution of the system of simultaneous equation you developed in part (b) using the matrix method.

## FORMULA SHEET

$$
\begin{array}{ll}
V=P(1+r n) & V=P(1+r)^{n} \\
V=P(1-r)^{n} & A_{O D I}=R\left\{\frac{(1+r)^{n}-1}{r}\right\} \\
P_{O D I}=R\left\{\frac{1-(1+r)^{-n}}{r}\right\} & \\
P_{P E R}=R\left\{\frac{1}{r}\right\} & \\
I R R=r_{1}+\left\{\frac{N P V_{1}}{N P V_{1}-N P V_{2}}\right\}\left(r_{2}-r_{1}\right) &
\end{array}
$$

$$
\begin{aligned}
& A^{-1}=\left(\frac{1}{|A|}\right) \operatorname{adj}(A) \\
& A X=b \Rightarrow X=A^{-1} b
\end{aligned}
$$

