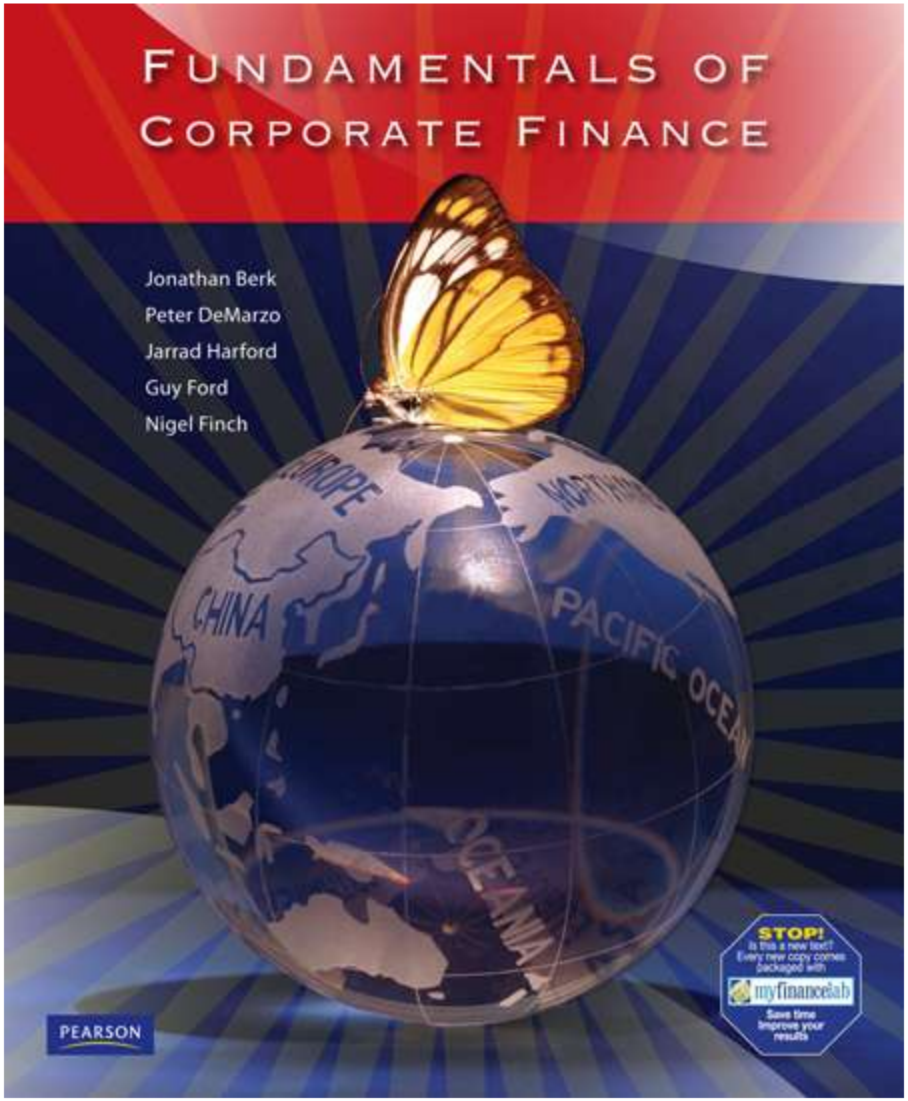
PowerPoint
to accompany



Chapter 5

Interest Rates

5.1 Interest Rate Quotes and Adjustments

- To understand interest rates, it's important to think of interest rates as a price—*the price of using money*.
- When you borrow money to buy a house, you are using the bank's money to purchase the house now and paying the money back over time.
- The interest rate on your loan is the *price you pay* to be able to convert your future loan payments into a house today.

5.1 Interest Rate Quotes and Adjustments

- **Annual percentage rates (APR)**
  - The APR is a way of quoting the actual interest earned each period—without the effect of compounding:

$$\text{Interest rate per compounding period} = \frac{APR}{m}$$

(m = number of compounding periods per year)

(Eq. 5.2)

FORMULA!

5.1 Interest Rate Quotes and Adjustments

- **The Effective Annual Rate (EAR)**
  - EAR is the total amount of interest that will be earned at the end of one year.
  - For example, with an EAR of *r* = 5%, a $100 investment grows to:

Month:	0	1	2
Cash Flow:	$100 x (1.05)	= $105 x (1.05)	= $110.25
	$100	x $(1.05)^2$	= $110.25
	$100	x (1.1025)	= $110.25

5.1 Interest Rate Quotes and Adjustments

- **Adjusting the discount rate to different time periods**
  - In general, by raising the interest rate factor (1 + *r*) to the appropriate power, we can calculate an equivalent interest rate for a longer (or shorter) time period.
  - For example, earning 5% interest in one year is equivalent to receiving $(1 + r)^{0.5} = (1.05)^{0.5} = \$1.0247$ for each $1 invested every six months (0.5 years).
  - A 5% EAR is equivalent to a rate of 2.47% every six months.

Month:	0		$\frac{1}{2}$		1
Cash flow:	$1	× (1.0247)	= $1.0247	× (1.0247)	= $1.05
	$1	×	$(1.0247)^2$	=	$1.05
	$1	×	(1.05)	=	$1.05

Adjusting the Discount Rate to Different Time Periods

- We can convert a discount rate of *r* for one period to an equivalent discount rate for *n* periods using the following formula:

$$\text{Equivalent } n\text{-period discount rate} = (1+r)^n - 1$$

FORMULA!

(Eq. 5.1)

Example 5.1
Valuing Monthly Cash Flows (pp.133-4)

Problem:

- Suppose your bank account pays interest monthly with an effective annual rate of **6%**.
- *What amount of interest will you earn each month?*

Execute:

- From Eq. 5.1, a **6%** EAR is equivalent to earning $(1.06)^{1/12} - 1 = 0.4868\%$ per month.
- The exponent in this equation is **1/12** because the period is **1/12th** of a year (a month).

5.1 Interest Rate Quotes and Adjustments

- **Converting an APR to an EAR**
  - Once we have calculated the interest earned per compounding period from Eq. 5.2, we can calculate the equivalent interest rate for any other time interval using Eq. 5.1.

$$1 + EAR = \left(1 + \frac{APR}{m}\right)^m$$

(m=number of compounding periods per year)

(Eq. 5.3)

FORMULA!

Table 5.1 Effective Annual Rates for a 6% APR with Different Compounding Periods

Compounding interval	Effective annual rate
Annual	$\left(1 + \frac{0.06}{1}\right)^1 - 1 = 6\%$
Semiannual	$\left(1 + \frac{0.06}{2}\right)^2 - 1 = 6.09\%$
Monthly	$\left(1 + \frac{0.06}{12}\right)^{12} - 1 = 6.1678\%$
Daily	$\left(1 + \frac{0.06}{365}\right)^{365} - 1 = 6.1831\%$

5.2 Application: Discount Rates and Loans

- Let’s apply the concept to solve two common financial problems:
  - calculating a *loan payment,* and
  - calculating the *remaining balance on a loan*.
- **Calculating loan payments**
  - Consider the timeline for a $30,000 car loan with these terms: 6.75% APR for 60 months. What is the instalment (C)?

Month:	0	1	2	60
Cash Flow	$30,000	- C	- C	- C

5.2 Application: Discount Rates and Loans

- Use the annuity formula to find *C*

- Note that 0.0675/12 = 0.005625

$$C = \frac{P}{\frac{1}{r}\left(1 - \frac{1}{(1+r)^n}\right)} = \frac{30\,000}{\frac{1}{0.005625}\left(1 - \frac{1}{(1+0.005625)^{60}}\right)} = \$590.50$$

Example 5.3 Calculating the Outstanding Loan Balance (p.139)

Problem:

- Let's say that you are now **three** years into your **$30,000** car loan (at 6.75% APR, for 60 months) and you decide to sell the car.
- When you sell the car, you will need to pay whatever the remaining balance is on your car loan.
- *After **36** months of payments, how much do you still owe on your car loan?*

Example 5.3 Calculating the Outstanding Loan Balance (p.139)

Solution:

Plan:

- We have already determined that the monthly payments on the loan are **$590.50**. The remaining balance on the loan is the present value of the remaining **two** years, or **24** months, of payments.
- Thus, we can just use the annuity formula with the monthly rate of **0.5625%**, a monthly payment of **$590.50**, and **24** months remaining.

Example 5.3 Calculating the Outstanding Loan Balance (p.139)

Execute:

$$\text{Balance with 24 months remaining} = \$590.50 \times \frac{1}{0.005625}\left(1 - \frac{1}{1.005625^{24}}\right) = \$13,222.32$$

- Thus, after **three** years, you owe **$13,222.32** on the loan.

Example 5.3 Calculating the Outstanding Loan Balance (p.139)

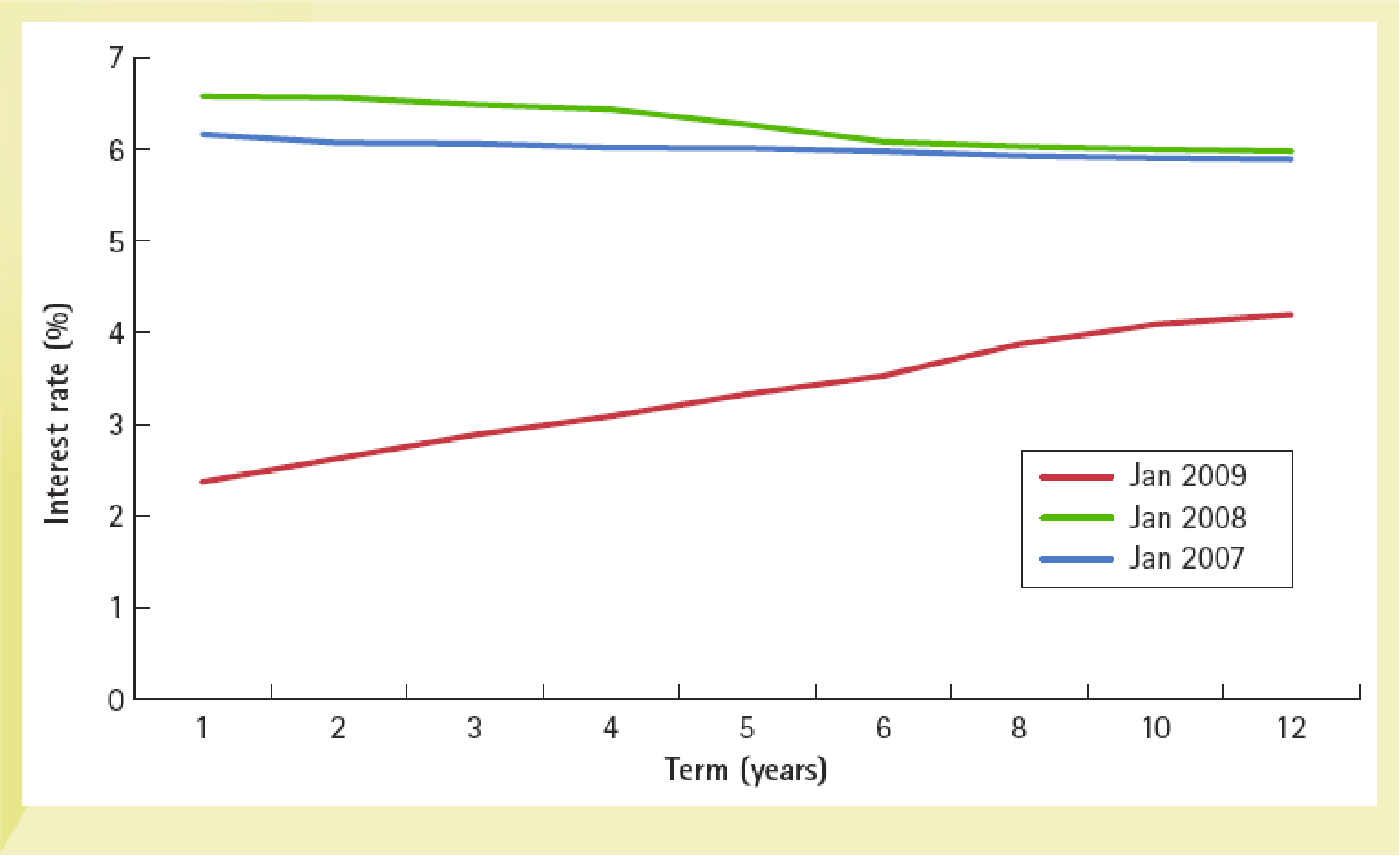
Evaluate:

- Any time that you want to end the loan, the bank will charge you a lump sum equal to the *present value* of your remaining payments.
- The amount you owe can also be thought of as the *future value* of the original amount borrowed after deducting payments made along the way.

5.3 The Determinants of Interest Rates

- The relationship between the investment term and the interest rate is called the **term structure** of interest rates.
- We can plot this relationship on a graph called the **yield curve.**
- Note that the interest rate depends on the horizon.
- The plotted rates are interest rates for Australian government bonds, which are considered as a risk-free interest rate.

Figure 5.3 Term Structure of Risk-Free Australian Interest Rates—Jan 2007, 2008 & 2009



Source: Data from Reserve Bank of Australia.

5.3 The Determinants of Interest Rates

- We can apply the term logic when calculating the PV of cash flows with different maturities.
- A risk-free cash flow of C_n received in n years has the present value:

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$$PV = \frac{C_n}{(1 + r_n)^n} \qquad \text{(Eq. 5.6)}$$

- where r_n is the risk-free interest rate for an n-year term.

5.3 The Determinants of Interest Rates

- ## PV of a cash flow stream
  - Combining Eq. (5.6) for cash flows in different years leads to the general formula for the present value of a cash flow stream:

$$PV = \frac{C_1}{1 + r_1} + \frac{C_2}{(1 + r_2)^2} + \cdots + \frac{C_N}{(1 + r_N)^N}$$

(Eq. 5.7)

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Example 5.5 Using the Term Structure to Calculate PVs (p.145)

Problem:

- Calculate the **present value** of a risk-free five-year annuity of **$1,000** per year, given the yield curve for January 2009 in **Figure 5.3**.
- The 1, 2, 3, 4 and 5-year interest rates in January 2009 were: 2.41%, 2.65%, 2.90%, 3.11% and 3.35%.

Period:	0	1	2	3	4	5
Cash Flow	0	$1,000	$1,000	$1,000	$1,000	$1,000

Example 5.5 Using the Term Structure to Calculate PVs (p.145)

Execute:

- To calculate the present value, we discount each cash flow by the corresponding interest rate:

$$PV = \frac{1000}{1.0241} + \frac{1000}{1.0265^2} + \frac{1000}{1.029^3} + \frac{1000}{1.0311^4} + \frac{1000}{1.0335^5} = 4,576$$

Example 5.5 Using the Term Structure to Calculate PVs (p.145)

Evaluate:

- The yield curve tells us the market interest rate per year for each different maturity.
- In order to correctly calculate the **PV** of cash flows from five different maturities, we need to use the five different interest rates corresponding to those maturities.
- *Note that we cannot use the annuity formula here because the discount rates differ for each cash flow.*