Investments: An Introduction

10e

Chapter 3: The Time Value of Money

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Time Value

- The process of expressing
  - the future value of cash flows (compounding)
  - the present value of future flows (discounting)

Time Value

- Payments are either
  - a single payment
  - a series of equal payments (an annuity)

Future Value

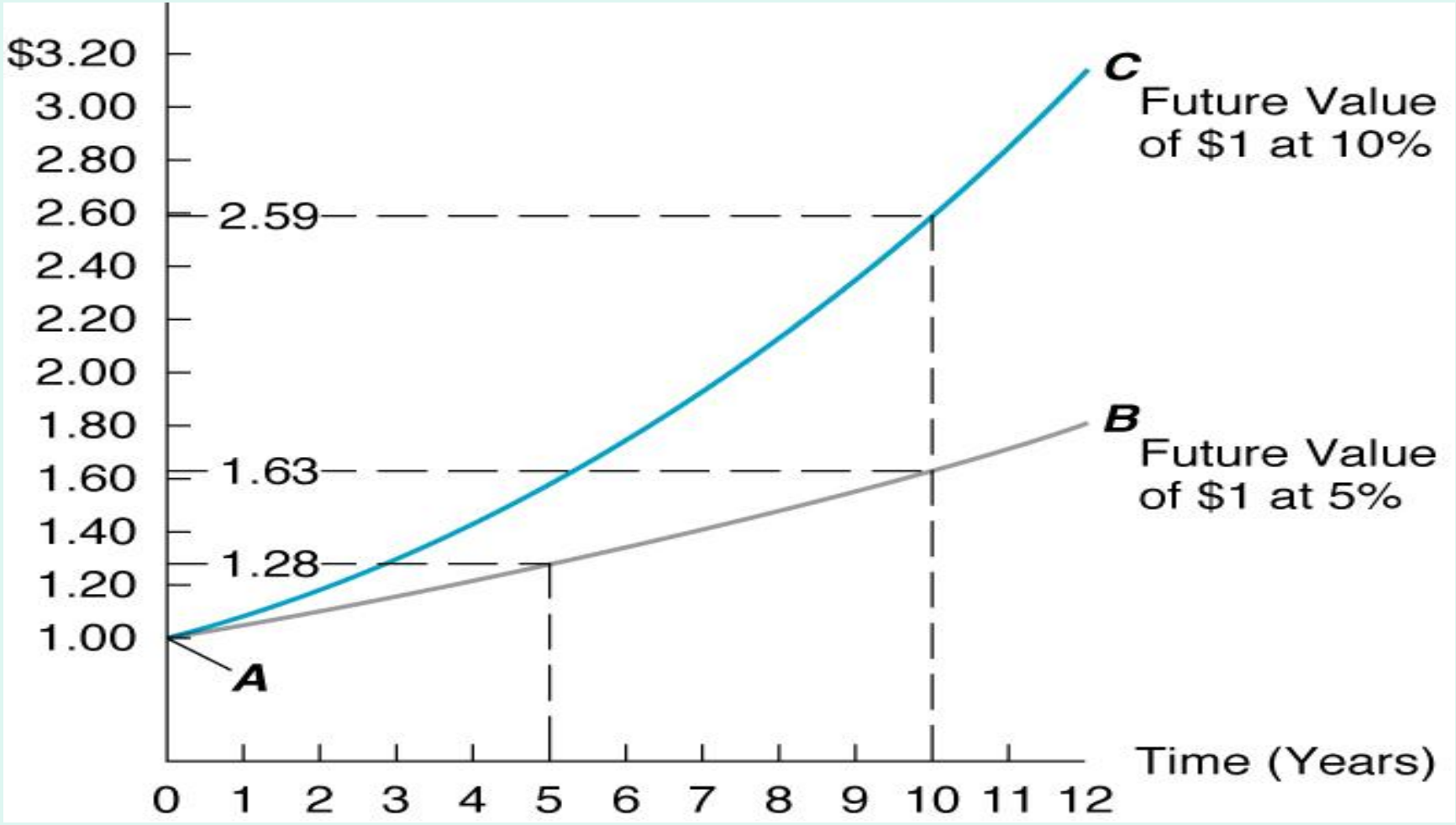
- The future value of $1 takes a single payment in the present into the future
- The general equation for the future value of $1:

$$P_0(1 + i)^n = P_n$$

Greater Terminal Values

- Higher interest rates
- Longer time periods
- Result in greater terminal values

Greater Terminal Values



Present Value

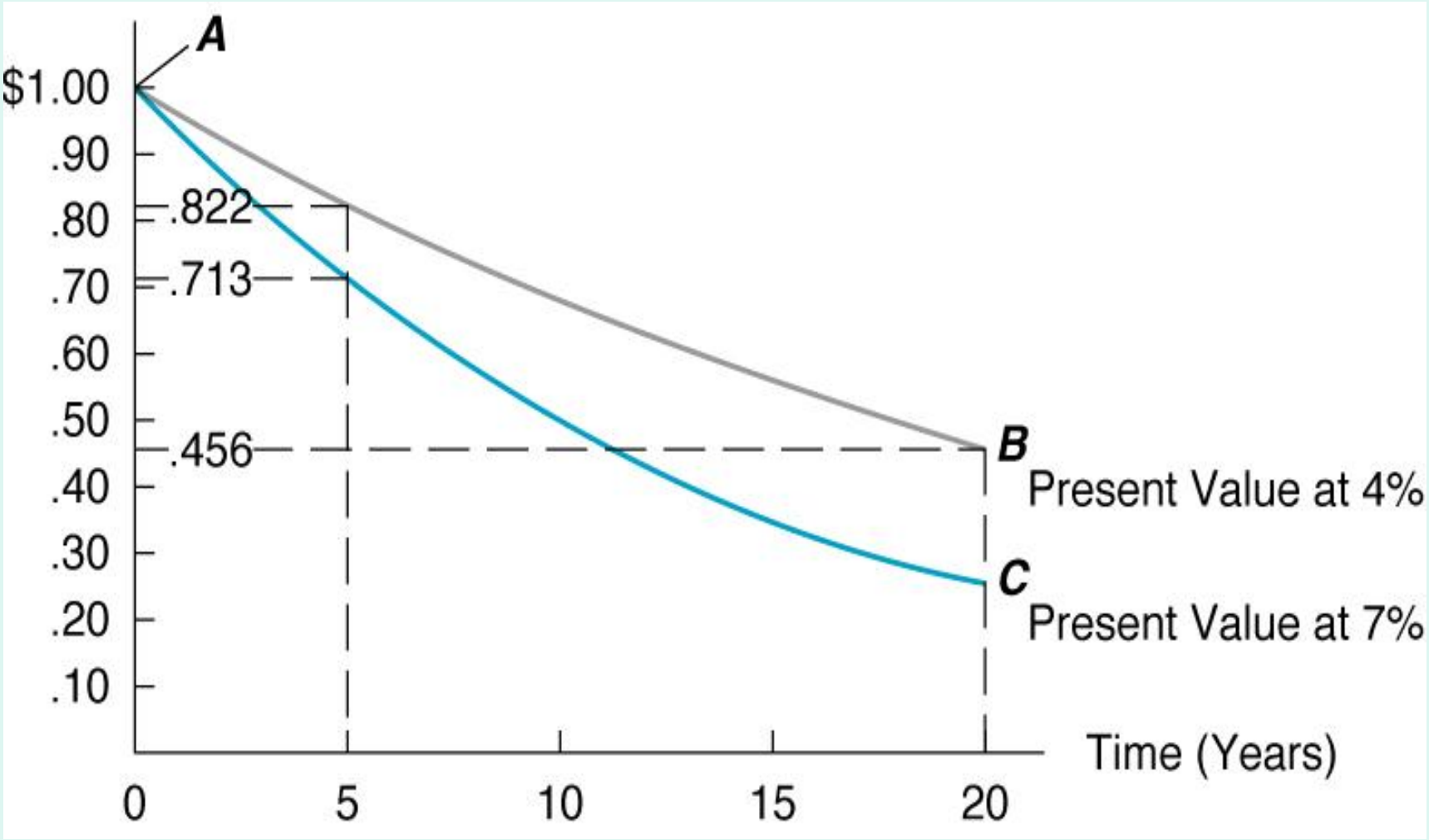
- The present value of $1 brings a single payment in the future back to the present.
- The general equation for the present value of $1:

$$P_0 = \frac{P_n}{(1+i)^n}$$

Lower Present Values

- Higher interest rates
- Longer time periods
- Result in lower present values

Lower Present Values



Annuities - Future Sum

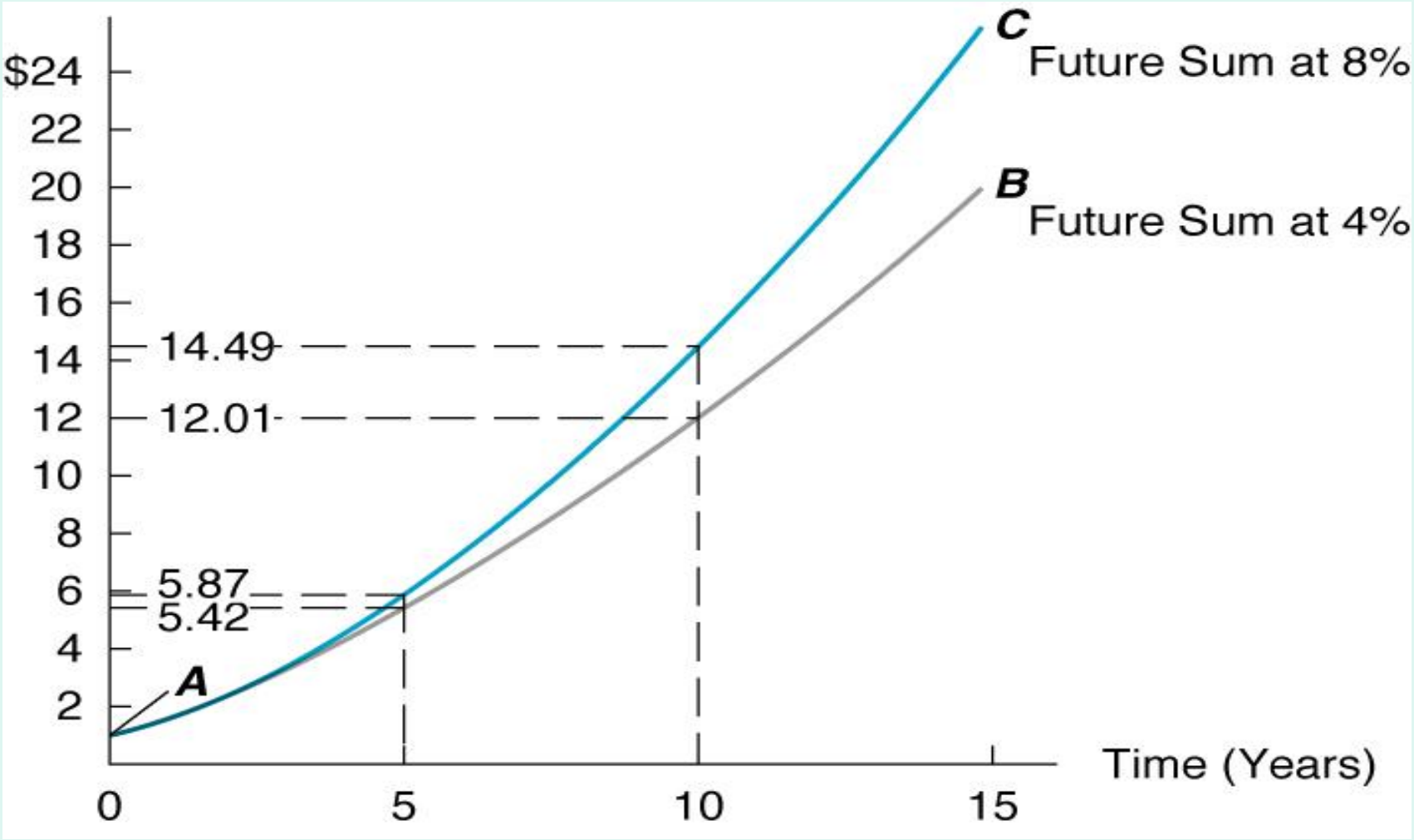
- The future sum of annuity takes a series of payments into the future.

- Payments may be made:
  - at the end of each time period (ordinary annuity), or
  - at the beginning of each time period (annuity due).

Greater Terminal Values

- Higher interest rates
- Longer time periods
- Result in greater terminal values

Greater Terminal Values



Annuities - Present Value

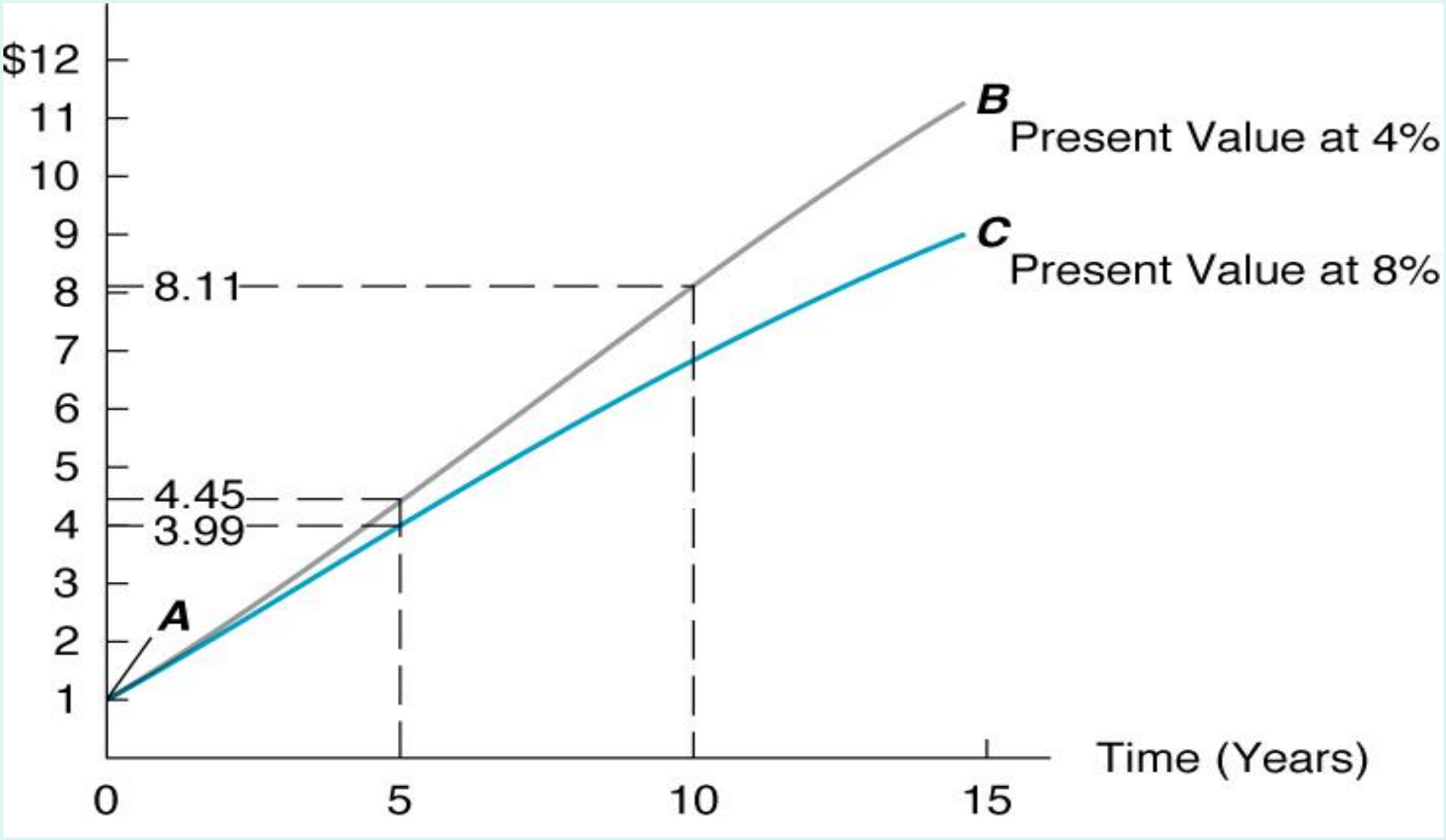
- The present value of an annuity brings a series of payments in the future back to the present.

Annuities - Present Value

- Higher interest rates result in lower present values.

- But longer time periods increase the present value (because more payments are received).

Annuities - Present Value



Time Value Problems

- Time value of money problems may be solved by using:
  - algebraic formulas
  - interest tables
  - financial calculators
  - software

Variables for Time Value of Money Problems

- PV = present value
- FV = future value
- PMT = annual payment
- N = number of time periods
- I = interest rate per period

Financial Calculators

- Express the cash inputs (PV, FV, and PMT) as cash inflows and cash outflows
- At least one of the cash variables must be
  - an inflow (+)
  - an outflow (-)

Future Value of $1: Illustration

- What is the future value after ten years of $1000 deposited in a savings account that pays 4 percent annually?

Future Value of $1

- The unknown: FV
- The givens:
  - PV = -1000
  - PMT = 0
  - N = 10
  - I = 4

The answer:
$1,480.24

Interpretation

- Depositing $1,000 (a current cash outflow) in the saving account produces $1,480.24 after ten years.
- Of the $1,480.24,
  - $1,000 is the initial principal, and
  - $480.24 is the earned interest.

Present Value of $1: Illustration

- What is the present value $1,480.24 received after ten years if the rate of interest is 4 percent annually?

Present Value of $1

- The unknown: PV
- The givens:
  - FV = 1,480.24
  - PMT = 0
  - N = 10
  - I = 4

The answer:
$1,000

Interpretation

- $1,480.24 received after ten years is worth $1,000 today if the interest rate is 4 percent.

Interpretation of Future and Present Values

These two problems are the same:

- In the first case, the $1,000 is compounded into its future value ($1,480.24).
- In the second case, the future value ($1,480.24) is discounted back to its present value ($1,000).

Future Value of an Ordinary Annuity: Illustration

- You deposit $1,000 in an account at the end of each year for ten years. What is the total amount in the account if you earn 8 percent annually?

Future Value of an Ordinary Annuity

- The unknown: FV
- The givens:
  - PV = 0
  - PMT = -1,000
  - N = 10
  - I = 8

The answer:
$14,486.56

Interpretation

- For an annual cash payment of $1,000, you will have $14,486.56 after ten years.
- Of the $14,486.56,
  - $10,000 is the total cash outflow, and
  - $4,486.56 is the earned interest.

Present Value of an Ordinary Annuity: Illustration

- What is the present value of (or required cash outflow to purchase) an ordinary annuity of $10,000 for ten years, if the rate of interest is 8 percent?

Present Value of an Annuity

- The unknown: PV
- The givens:
  - FV = 0
  - PMT = 10,0000
  - N = 10
  - I = 8

The answer:
$67,100.81

Interpretation

- For a present payment of $67,100.81, the individual will annually receive $10,000 for the next ten years.
  - The $67,100.81 is an immediate cash outflow;
  - The $10,000 annual payment is a cash inflow.